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QUESTION BANK (DESCRIPTIVE)

Subject with Code: Modern Control Theory (18EE0223)

Course & Branch: B.Tech - EEE

Year & Sem: III-B.Tech & II-Sem

UNIT –I

Regulation: R18

STATE VARIABLE DESCRIPTION AND SOLUTION OF STATE EQUATION

- 1 a) Define state variable. [L1][CO1] [2M]
 b) Write any two properties of state transition matrix. [L1][CO1] [2M]
 c) What are the advantages of state space representation? Compare with transfer function representation. [L1][CO1] [2M]
 d) What is state diagram? [L1][CO1] [2M]
 e) Define state model. [L1][CO1] [2M]
- 2 Consider the following transfer function of a system $\frac{y(s)}{U(s)} = \frac{s+6}{s^2+6s+6}$. [L1][CO1] [10M]
 Obtain state space representation of the system.
- 3 a) Explain state model and prove state model representation is not unique with example. [L2][CO1] [5M]
 b) Construct a state model for a system characterized by the differential equation $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = \frac{d^3u}{dt^3} + 8\frac{d^2u}{dt^2} + 17\frac{du}{dt} + 8u$. [L6][CO1] [5M]
- 4 a) Derive a solution of homogeneous state equation. [L3][CO1] [5M]
 b) Obtain the state transition matrix of $A = \begin{bmatrix} -6 & 1 & 0 \\ -11 & 0 & 1 \\ -6 & 0 & 0 \end{bmatrix}$. [L1][CO1] [5M]
- 5 a) State and prove the various properties of state transition matrix. [L5][CO1] [5M]
 b) Computer the solution of state equation $\dot{X} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U; X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ [5M]
- 6 Obtain state space representation for following systems [L1][CO1] [10M]
 (a) $\frac{Y(s)}{U(s)} = \frac{10(s+4)}{s(s+1)(s+3)}$ (b) $\frac{Y(s)}{U(s)} = \frac{10}{s^3+4s^2+2s+1}$
- 7 Obtain a state space equation and output equation for the system [L1][CO1] [10M]
 defined by $\frac{Y(s)}{U(s)} = \frac{2s^3+s^2+s+2}{s^3+4s^2+5s+2}$
- 8 For a system represented by state equation $\dot{x}(t) = Ax(t)$. The response is $x(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix}$ when $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ & $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ where $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. [L5][CO1] [10M]
 Determine the system matrix A and the state transition matrix.
- 9 $X = \begin{bmatrix} -1 & -4 & -1 \\ -1 & -6 & -2 \\ -1 & -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} U; Y = [1 \ 1 \ 1]X$. Find the transfer function of the system. [L1][CO1] [10M]
- 10 Explain state model and derive the solutions of state equation. [L2][CO1] [10M]

CONTROLLABILITY, OBSERVABILITY

- 1 a) Define controllability. [L1][CO2] [2M]
 b) What is need for observability test? [L1][CO2] [2M]
 c) State the reality between controllability and observability. [L2][CO2] [2M]
 d) State the condition for observability by Kalman's method. [L2][CO2] [2M]
 e) What canonical form of state model? [L1][CO2] [2M]
- 2 a) Define Controllability. What are the tests to find the controllability of a system? [L1][CO2] [5M]
 b) The state equation is given by $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} U$. Test for controllability. [L4][CO2] [5M]
- 3 a) Define Observability. What are the tests to find the observability of a given system? [L1][CO2] [5M]
 b) Test observability for $\dot{x}_1 = -2x_1 + x_2 + U$, $\dot{x}_2 = -x_2 + U$ and $y = x_1 + x_2$. [L4][CO2] [5M]
- 4 A System is represented by the state model: [L1][CO2] [10M]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} U; y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Check whether system is (a) Completely Controllable
 (b) Completely Observable.

- 5 A System is represented by the state model: [L4][CO2] [10M]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} U(t); y(t) = [4 \quad 6 \quad 8] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \quad \text{Test}$$

whether the system is (a) Completely Controllable
 (b) Completely Observable

- 6 Consider the system $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ the output is given by [L1][CO2] [10M]

$$Y = [1 \quad 1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(a) Show that the system is not completely observable
 (b) Show that the system is completely observable if the output is given

$$\text{by } \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

- 7 State and prove the principle of duality between controllability and observability. [L2][CO2] [10M]
- 8 The state model of a system is given by [L2][CO2] [10M]
- $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} [u]; Y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$
- Convert the state model to canonical form
- 9 The state model of a system is given by [L1][CO2] [10M]

$$\dot{x} = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix} x + \begin{bmatrix} 11 \\ 1 \\ -14 \end{bmatrix} u; Y = [-3 \quad -5 \quad -2]x. \text{ Find the canonical format representation.}$$

- 10 Write the effect of state feedback on controllability and observability. [L1][CO2] [10M]

UNIT -III**STATE FEEDBACK CONTROLLERS AND OBSERVERS**

- 1 a) What is pole placement by state feedback? [L1][CO3] [2M]
 b) Define state observer? [L1][CO3] [2M]
 c) What is the need for state observer? [L1][CO3] [2M]
 d) Define full order & reduced order observer. [L1][CO3] [2M]
 e) What is the necessary condition to be satisfied for design of state observer? [L1][CO3] [2M]
- 2 Explain the design of pole placement controller using state feedback. [L1][CO3] [10M]
- 3 Consider a linear system described by the transfer function $\frac{Y(s)}{U(s)} = \frac{10}{s(s+1)(s+2)}$. Design a feedback controller with a state feedback, so that the closed loop poles are placed at -2, $-1 \pm j1$. [L1][CO3] [10M]
- 4 A single input system is described by the following state equation [L1][CO3] [10M]
- $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 0 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 10 \\ 1 \\ 0 \end{bmatrix} U.$$
- Design a state feedback controller which will give closed loop poles at $-1 \pm j2, 6$.
- 5 Explain the full order and reduced order observer. [L1][CO3] [10M]
- 6 As $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} U; Y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Convert the state model to observable phase variable form. [L2][CO3] [10M]
- 7 The state model is given by [L2][CO3] [10M]
- $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} U; Y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$
- Convert the state model to controllable phase variable form.
- 8 Consider the system described by the state model $\dot{x} = Ax; y = Cx$; Where $A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}; C = [1 \ 0]$. Design a full order state observer. The desired eigen values for the observer matrix are $\mu_1 = -5; \mu_2 = -5$. [L1][CO3] [10M]
- 9 What is state observer? Explain about state observer. [L1][CO3] [10M]
- 10 Consider the system defined by [L1][CO3] [10M]
- $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t); y(t) = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$
- Design a full order state observer assuming the desired poles for the observer are located at -10, -10, -15.

UNIT -IV

ANALYSIS OF NON LINEAR SYSTEMS

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|----|----|--|-----------|-------|
| 1 | a) | How nonlinearities are introduced in the system. | [L1][CO4] | [2M] |
| | b) | What are the methods available for the analysis of nonlinear system? | [L1][CO4] | [2M] |
| | c) | What is dead zone? | [L1][CO4] | [2M] |
| | d) | What is phase trajectory? | [L1][CO4] | [2M] |
| | e) | How limit cycles are determined from phase portrait. | [L1][CO4] | [2M] |
| 2 | | Derive the describing function of back lash nonlinearities. | [L6][CO4] | [10M] |
| 3 | | Derive the describing function of saturation nonlinearities. | [L6][CO4] | [10M] |
| 4 | | Derive the describing function of relay with dead zone. | [L6][CO4] | [10M] |
| 5 | | Explain the classification of non-linear systems. | [L2][CO4] | [10M] |
| 6 | | With the help of graphical representations, explain about various common physical nonlinearities. | [L2][CO4] | [10M] |
| 7 | | Explain the method of isoclines for the construction of phase trajectories. | [L2][CO4] | [10M] |
| 8 | | What is singular point? Explain various types of singular points. | [L1][CO4] | [10M] |
| 9 | | A linear second order servo is described by the equation
$\ddot{e} + 2\zeta\omega_n \dot{e} + \omega_n^2 e = 0$ Where, $\zeta = 0.15$, $\omega_n = 1$ rad/sec, $e(0) = 1.5$
and $\dot{e}(0) = 0$. Determine the singular point construct the phase trajectory using method of isoclines. | [L5][CO4] | [10M] |
| 10 | a) | Explain in detail about various characteristics of non-linear systems. | [L2][CO4] | [5M] |
| | b) | Describe various types of singular points and their corresponding phase portraits with rough sketches | [L1][CO4] | [5M] |

UNIT – V
STABILITY ANALYSIS

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|---|----|--|-----------|-------|
| 1 | a) | State Lyapunov instability theorem. | [L5][CO5] | [2M] |
| | b) | State Lyapunov stability theorem. | [L5][CO5] | [2M] |
| | c) | What is the condition for stability in Lyapunov direct method? | [L1][CO5] | [2M] |
| | d) | What are the linear autonomous system? | [L1][CO5] | [2M] |
| | e) | Define positive definiteness of a system. | [L1][CO5] | [2M] |
| 2 | | State and prove Lyapunov stability theorem | [L5][CO5] | [10M] |
| 3 | | Show that the asymptotically stable condition of a linear system $\dot{x} = Ax$ at origin is: $A^T P + PA = -Q$. Where P&Q are the symmetric positive definite matrices. | [L2][CO5] | [10M] |
| 4 | | Consider the non-linear system: $\dot{x}_1 = x_2, \dot{x}_2 = -x_1 - x_1^2 x_2$ investigate the stability of this non-linear system around its equilibrium point at origin. | [L4][CO5] | [10M] |
| 5 | | Use Krasovskii's theorem to show that the equilibrium state $x=0$ of the system described by $\dot{x}_1 = -3x_1 + x_2, \dot{x}_2 = x_1 - x_2 - x_2^3$ is asymptotically stable in the large. | [L2][CO5] | [10M] |
| 6 | a) | State and prove Lyapunov instability theorem. | [L5][CO5] | [5M] |
| | b) | Show that the following quadratic form is positive definite $V(x) = 8x_1^2 + x_2^2 + 4x_3^2 + 2x_1x_2 - 4x_1x_3 - 2x_2x_3$ | [L1][CO5] | [5M] |
| 7 | a) | Show the graphical representation of stability, asymptotic stability and | [L1][CO5] | [5M] |

- instability
- b) Define quadratic form and Hermitian form. [L1][CO5] [5M]
- 8 Using Lyapunov analysis, determine the stability of the equilibrium state [L5][CO5] [10M]
of the system $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.
- 9 Examine the stability of the system described by the following equation [L4][CO5] [10M]
by Krasovskii's theorem $x'_1 = -x_1$ $x'_2 = x_1 - x_2 - x_2^3$
- 10 State and explain about Lyapunov stability for non-linear system. [L2][CO5] [10M]

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